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A Smooth Penalty Function Approach for Constrained Crop Planning Optimization

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Abstract. Agricultural planning requires the efficient allocation of limited resources such as land, water, and capital among competing crops. Such decision-making problems are naturally constrained and may become nonlinear when yield response, production cost, and environmental effects are included. This paper presents a smooth penalty function approach for a constrained crop planning optimization model. The aim is to maximize agricultural profit while satisfying land availability, water requirement, budget, and non-negativity restrictions. The constrained problem is transformed into a sequence of smooth bound-constrained optimization problems by replacing the positive part of each constraint violation with a continuously differentiable approximation. The resulting formulation is suitable for gradient-based and quasi-Newton algorithms and avoids the nonsmooth behavior associated with classical exact penalty functions. A practical algorithm based on gradually increasing penalty parameters is described. A numerical crop planning example involving wheat, rice, and maize is used to illustrate the method. The corrected computational results show that the optimal allocation is feasible with respect to land, water, and budget restrictions, and that the water and land constraints become active at the optimum. The study indicates that smooth penalty methods provide a flexible and numerically stable framework for resource allocation problems in agriculture.

Keywords: Smooth penalty function, constrained optimization, crop planning, nonlinear programming, resource allocation, agricultural optimization.

INTRODUCTION

Optimization techniques play an important role in agricultural planning and resource management. Farmers, planners, and policy makers often need to allocate limited resources such as land, irrigation water, labor, and capital among several crops. The quality of this allocation directly affects profitability, food production, and sustainability. Since agricultural resources are finite and crop requirements are heterogeneous, crop planning is naturally formulated as a constrained optimization problem.

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Classical agricultural optimization models have frequently used linear programming and nonlinear programming for crop selection, irrigation scheduling, fertilizer management, and farm planning. These models are useful because they provide a clear mathematical structure for decision making. However, real agricultural systems may involve nonlinear yield response, changing market prices, uncertain water availability, and environmental restrictions. In such cases, the direct treatment of constraints can become computationally difficult, especially when several constraints interact simultaneously.

Penalty function methods offer a systematic way to handle constraints by incorporating violations into the objective function. Instead of solving the constrained problem directly, one solves a sequence of unconstrained or bound-constrained problems. Classical penalty methods are simple and widely applicable, but nonsmooth penalty terms may restrict the use of efficient gradient-based methods. Furthermore, very large penalty parameters can lead to ill-conditioned subproblems.

Smooth penalty functions address these limitations by replacing nonsmooth violation measures with differentiable approximations. This preserves the basic logic of penalty methods while allowing the use of gradient-based and quasi-Newton optimization algorithms. Such methods are especially attractive for agricultural optimization, where models may be extended to include nonlinear production response, risk, and sustainability criteria.

The present paper applies a smooth penalty function approach to a crop planning model. The objective is to determine the area allocated to each crop so that total profit is maximized subject to land, water, and budget constraints. The contribution of the paper is twofold: first, it gives a clear smooth penalty formulation for the constrained crop allocation problem; second, it presents a corrected numerical example with graphical interpretation of the optimal allocation and resource utilization.

REVIEW OF LITERATURE

Penalty function methods are among the classical techniques for constrained optimization. Zangwill introduced early penalty approaches for nonlinear programming. Fiacco and McCormick developed the sequential unconstrained minimization technique, which became an important foundation for penalty-based algorithms. Exact penalty functions were studied by Fletcher and by Han and Mangasarian, while global convergence properties were investigated in later works on nonlinear programming and convex optimization.

A common difficulty with exact penalty functions is nonsmoothness. Since many efficient optimization algorithms require differentiability, smoothing techniques were developed to

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approximate nonsmooth penalty terms. Pinar and Zenios studied smoothing strategies for exact penalty functions, and subsequent work extended smooth penalty ideas to broader nonlinear programming settings. Modern treatments of nonlinear programming also discuss penalty, barrier, and augmented Lagrangian methods as standard tools for constrained optimization.

In agricultural planning, mathematical programming has been widely used for resource allocation and farm decision making. Early agricultural programming models focused mainly on profit maximization under resource restrictions. Later studies introduced goal programming, multi-objective optimization, sustainability measures, and decision-support frameworks. Despite these developments, the application of smooth penalty techniques to crop planning problems remains relatively limited. This motivates the present study, which connects smooth penalty methodology with a practical crop allocation model.

MATHEMATICAL FORMULATION OF THE CROP PLANNING PROBLEM

Consider a farm planning problem involving n crops. Let

$$x_i \geq 0, \qquad i = 1,2,\dots,n,$$

denote the area, in hectares, allocated to crop i. The total available land is denoted by L.

The profit per hectare from crop i is

$$r_i = p_i y_i - c_i,$$

where p_i is the market price per unit yield, y_i is the expected yield per hectare, and c_i is the cultivation cost per hectare. The total profit is therefore

$$f(x) = \sum_{i=1}^{n} r_i x_i.$$

The crop planning problem is written as

$$\max_{x} \quad f(x) = \sum_{i=1}^{n} r_i x_i$$

$$subject\ to \quad \sum_{i=1}^{n} x_i \leq L,$$

$$\sum_{i=1}^{n} w_i x_i \leq W,$$

$$\sum_{i=1}^{n} c_i x_i \leq B,$$

$$x_i \geq 0, \quad i = 1,2, \dots, n. \qquad (1)$$

where w_i is the water requirement per hectare for crop i, W is the total available water, and B is the available budget.

For compact notation, define the inequality constraints

$$g_1(x) = \sum_{i=1}^{n} x_i - L, g_2(x) = \sum_{i=1}^{n} w_i x_i - W, g_3(x) = \sum_{i=1}^{n} c_i x_i - B.$$

The feasible region is determined by $g_j(x) \leq 0$, $j = 1,2,3$, together with $x \geq 0$.

SMOOTH PENALTY FUNCTION APPROACH

For the constrained maximization problem

$$max\, f(x) \quad subject\ to \quad g_j(x) \leq 0, \quad j = 1,2, \dots, m,$$

a classical exterior penalty formulation is

$$\Phi(x, \mu) = f(x) - \mu \sum_{j=1}^{m} max\{0, g_j(x)\}^2,$$

where $\mu > 0$ is a penalty parameter. Although this formulation penalizes infeasible points, the function $max\{0, g_j(x)\}$ is not differentiable at $g_j(x) = 0$.

To obtain a smooth formulation, the positive-part function is approximated by the softplus function

$$\psi_\alpha(t) = \frac{1}{\alpha} ln(1 + e^{\alpha t}), \qquad \alpha > 0. \qquad (2)$$

The function ψ_α is continuously differentiable and satisfies

$$\lim_{\alpha \to \infty} \psi_\alpha(t) = max\{0, t\}.$$

Its derivative is

$$\psi_\alpha{}'(t) = \frac{1}{1 + e^{-\alpha t}},$$

which is smooth for every finite α.

For numerical stability, it is useful to work with normalized constraints

$$h_j(x) = \frac{g_j(x)}{s_j},$$

where $s_j > 0$ is a suitable scaling factor, such as L, W, or B. The smooth penalized objective is then

$$\Phi(x, \mu, \alpha) = f(x) - \mu \sum_{j=1}^{m} \left[\psi_\alpha\left(h_j(x)\right)\right]^2. \qquad (3)$$

The bound-constrained subproblem is

$$\max_{x \geq 0} \Phi(x, \mu, \alpha).$$

Algorithm

The smooth penalty method may be implemented as follows.

1. Choose an initial point $x^{(0)} \geq 0$, an initial penalty parameter $\mu_0 > 0$, a smoothing parameter $\alpha > 0$, a multiplier $\tau > 1$, and a tolerance $\varepsilon > 0$.
2. For fixed μ_k and α, solve

$$\max_{x \geq 0} \Phi(x, \mu_k, \alpha)$$

   using a gradient-based or quasi-Newton method.
3. If $g_j\left(x^{(k)}\right) \leq \varepsilon$ for all j, stop.
4. Otherwise, set $\mu_{k+1} = \tau \mu_k$, use the current solution as the next starting point, and repeat.

Under standard regularity assumptions, the solutions of the smooth penalized subproblems approach a feasible stationary point of the original constrained problem as the penalty parameter is increased and the smoothing error is reduced.

NUMERICAL EXPERIMENTS AND RESULTS

A representative crop planning example involving wheat, rice, and maize is considered. The economic and resource parameters are shown in Table I.

TABLE I. Crop data used in the numerical experiment

Crop	Yield (t/ha)	Price (Rs./t)	Cost (Rs./ha)	Water (m^3/ha)
Wheat	3.2	2200	1200	450
Rice	4.1	2500	1500	1200
Maize	3.8	2100	1100	600

The available resources are

$$L = 100\ ha, \qquad W = 70{,}000\ m^3, \qquad B = 130{,}000\ Rs.$$

The corresponding net profit per hectare is Rs. 5840 for wheat, Rs. 8750 for rice, and Rs. 6880 for maize.

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TABLE II. Optimal crop allocation

Crop	Optimal area (ha)
Wheat	0.00
Rice	16.67
Maize	83.33

TABLE III. Resource utilization at the optimal solution

Resource	Used	Available	Utilization
Land (ha)	100.00	100.00	100.00%
Water (m^3)	70,000.00	70,000.00	100.00%
Budget (Rs.)	116,666.67	130,000.00	89.74%

Solving the constrained model gives the optimal allocation shown in Table II. This corrected allocation satisfies all resource constraints. In particular, the water constraint is active at the optimum.

The maximum profit obtained at this allocation is approximately

$$f(x^*) = Rs. 719{,}166.67.$$

The resource consumption at the optimal solution is summarized in Table III.

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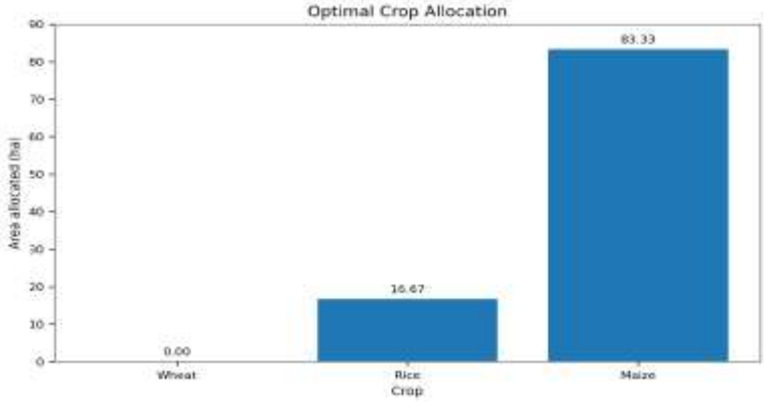

FIGURE 1. Optimal land allocation among wheat, rice, and maize.

Figures 1 and 2 give a graphical interpretation of the result. The solution assigns most of the land to maize because maize provides a strong profit return while using substantially less water than rice. Rice is still included because of its high net profit per hectare, but its large water requirement limits its optimal area. Wheat is not selected in this representative data set because, relative to maize and rice, it does not improve the optimal profit under the active land and water restrictions.

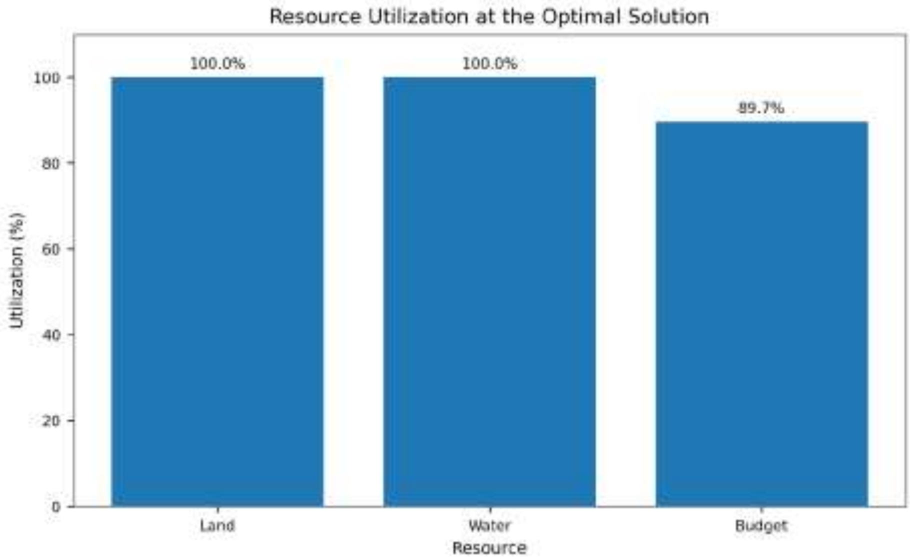

FIGURE 2. Percentage utilization of land, water, and budget at the optimal solution.

The resource utilization graph shows that land and water are fully used, whereas the budget constraint remains slack. Thus, for the chosen data, water availability is the main limiting factor after land is fully allocated. This is an economically meaningful outcome in crop planning, especially in regions where irrigation water is scarce.

CONCLUSIONS AND FUTURE WORK

This paper presented a smooth penalty function approach for a constrained crop planning optimization problem. The method transforms the original constrained problem into a sequence of smooth bound-constrained subproblems by using a differentiable approximation of constraint violations. This allows the use of efficient gradient-based and quasi-Newton methods while avoiding the nonsmoothness of classical penalty functions.

A numerical example involving wheat, rice, and maize was studied. The corrected optimal allocation satisfies all constraints and gives a maximum profit of approximately Rs. 719,166.67. The graphical results show that land and water are fully utilized, while the budget constraint remains nonbinding. The example demonstrates that smooth penalty methods can support practical agricultural decision making by producing feasible and economically interpretable crop allocations.

Future work may extend the model by incorporating nonlinear yield response functions, uncertain market prices, stochastic water availability, and environmental sustainability constraints. Multi-objective formulations that balance profit, water conservation, and soil health would also be useful. Another promising direction is to compare smooth penalty methods with interior-point, augmented Lagrangian, and metaheuristic approaches on larger agricultural data sets.

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