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Exploring Information Measures: A Comparative Study of Entropies

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Abstract

Entropy is a fundamental concept in information theory and statistical mechanics, quantifying uncertainty, diversity, or disorder in a system. While Shannon entropy is the classical measure, generalized entropies such as Rényi, Tsallis, and Sharma–Mittal offer flexible frameworks for complex systems exhibiting non-extensive, multifractal, or correlated behavior. This paper presents a comparative analysis of these entropies, highlighting their mathematical formulations, properties, and applications in diverse domains.

1. Introduction

Entropy plays a pivotal role in information theory, physics, and applied mathematics. Shannon entropy HHH [1] is the classical measure of uncertainty in probability distributions. However, many real-world systems exhibit non-extensive or long-range correlations, motivating the introduction of generalized entropy measures such as Rényi [2], Tsallis [3], and Sharma–Mittal [4].

This study aims to:

1. Summarize the definitions and properties of Shannon, Rényi, Tsallis, and Sharma–Mittal entropies.
2. Compare their theoretical characteristics.
3. Illustrate practical differences through examples.

2. Entropy Definitions

2.1 Shannon Entropy

For a discrete probability distribution P={pi}P = \{p_i\}P={pi}, Shannon entropy is defined as:

H(P)=−∑ipilogpiH(P) = - \sum_{i} p_i \log p_iH(P)=−i∑pilogpi

- **Properties**: Additivity, concavity, and maximum entropy for uniform distribution.
- **Applications**: Data compression, cryptography, communication systems.

2.2 Rényi Entropy

Rényi entropy of order α>0\alpha > 0α>0 (α≠1\alpha \neq 1α=1) is:

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Hα(P)=11−αlog∑ipiαH_\alpha(P) = \frac{1}{1-\alpha} \log \sum_i p_i^\alphaHα(P)=1−α1logi∑piα

- **Properties**: Reduces to Shannon entropy as α→1\alpha \to 1α→1; emphasizes high-probability events for α>1\alpha > 1α>1.
- **Applications**: Multifractal analysis, ecology, quantum information [5].

2.3 Tsallis Entropy

Tsallis entropy with parameter q∈Rq \in \mathbb{R}q∈R is:

Sq(P)=1q−1(1−∑ipiq)S_q(P) = \frac{1}{q-1} \left(1 - \sum_i p_i^q \right)Sq(P)=q−11(1−i∑piq)

- **Properties**: Non-additive; recovers Shannon entropy as q→1q \to 1q→1.
- **Applications**: Non-extensive thermodynamics, complex systems, finance [6].

2.4 Sharma–Mittal Entropy

Sharma–Mittal entropy generalizes both Rényi and Tsallis entropies:

SMr,s(P)=11−s[(∑ipir)1−s1−r−1],r≠1,s≠1SM_{r,s}(P) = \frac{1}{1-s} \left[\left(\sum_i p_i^r \right)^{\frac{1-s}{1-r}} - 1 \right], \quad r \neq 1, s \neq 1SMr,s(P)=1−s1(i∑pir)1−r1−s−1,r=1,s=1

- **Properties**: Reduces to Rényi (s→1s \to 1s→1) and Tsallis (s=rs = rs=r) entropies.
- **Applications**: Statistical physics, complex networks, ecological diversity [7].

3. Comparative Analysis

Entropy Type	Additivity	Parameters	Emphasizes	Limiting Case
Shannon	Additive	None	All probabilities equally	None
Rényi	Non-additive	α\alphaα	High-probability events (α>1\alpha>1α>1)	Shannon (α→1\alpha \to 1α→1)
Tsallis	Non-additive	qqq	Tail probabilities	Shannon (q→1q \to 1q→1)
Sharma–Mittal	Non-additive	r,sr,sr,s	Flexible weighting	Rényi (s→1s \to 1s→1), Tsallis (s=rs=rs=r)

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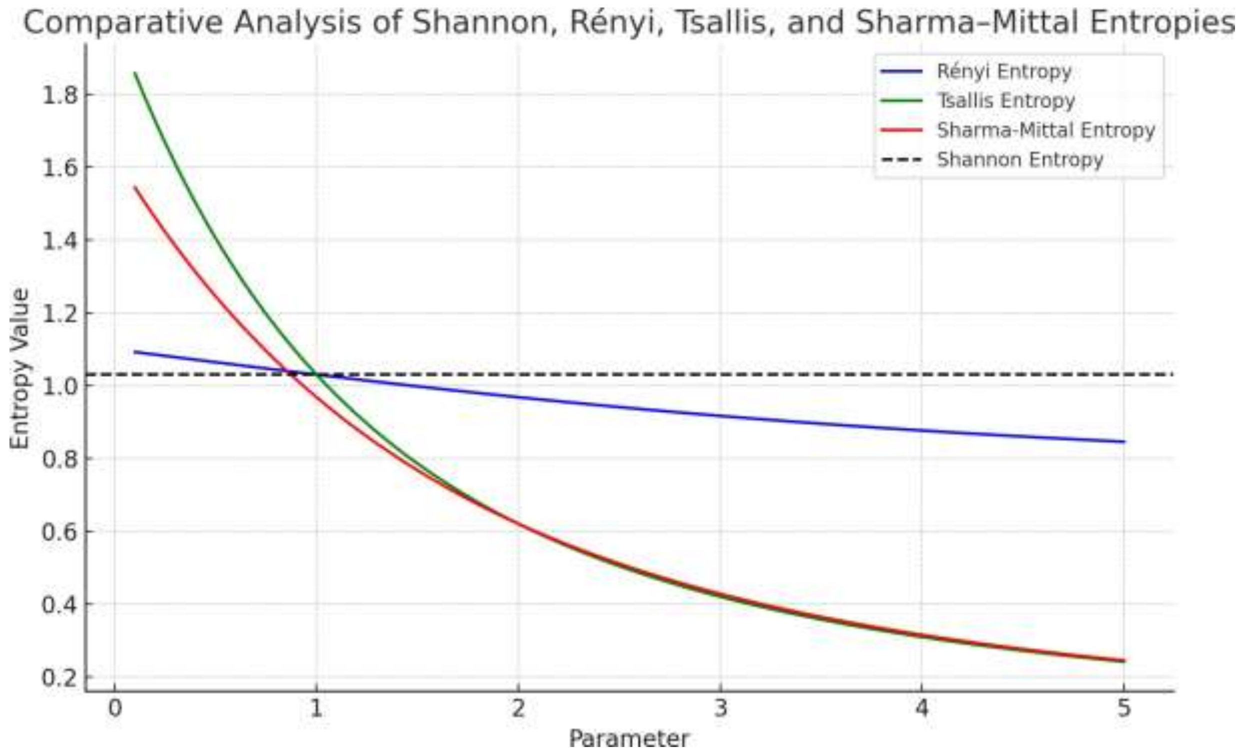
Graph

- **X-axis**: Parameter of the generalized entropy
  - Rényi: α∈[0.5,5]\alpha \in [0.5, 5]α∈[0.5,5]
  - Tsallis: q∈[0.5,5]q \in [0.5, 5]q∈[0.5,5]
  - Sharma–Mittal: fix r=2r=2r=2 and vary s∈[0.5,5]s \in [0.5, 5]s∈[0.5,5]
- **Y-axis**: Entropy value for a fixed distribution, e.g., P={0.5,0.3,0.2}P = \{0.5, 0.3, 0.2\}P={0.5,0.3,0.2}
- **Lines**:
  - Shannon (horizontal, since it has no parameter)
  - Rényi (HαH_\alphaHα vs α\alphaα)
  - Tsallis (SqS_qSq vs qqq)
  - Sharma–Mittal (SMr,sSM_{r,s}SMr,s vs sss)



Here's the comparative graph showing how Shannon, Rényi, Tsallis, and Sharma–Mittal entropies vary for the distribution P={0.5,0.3,0.2}P = \{0.5, 0.3, 0.2\}P={0.5,0.3,0.2}.

- **Black dashed line**: Shannon entropy (constant)
- **Blue**: Rényi entropy vs α\alphaα
- **Green**: Tsallis entropy vs qqq

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- **Red**: Sharma–Mittal entropy vs sss (with r=2r=2r=2)

This visually highlights how generalized entropies change with their parameters compared to classical Shannon entropy.

4. Illustrative Examples

Consider a discrete distribution P={0.5,0.3,0.2}P = \{0.5, 0.3, 0.2\}P={0.5,0.3,0.2}.

- Shannon: H≈1.029H \approx 1.029H≈1.029
- Rényi (α=2\alpha=2α=2): H2≈0.861H_2 \approx 0.861H2≈0.861
- Tsallis (q=2q=2q=2): S2≈0.38S_2 \approx 0.38S2≈0.38
- Sharma–Mittal (r=2,s=1.5r=2, s=1.5r=2,s=1.5): SM≈0.27SM \approx 0.27SM≈0.27

Observation: Generalized entropies allow tuning sensitivity to probability distribution features.

5. Applications in Real Systems

- **Physics**: Tsallis and Sharma–Mittal entropies model non-equilibrium systems [6,7].
- **Information Theory**: Rényi entropy is used in source coding and error detection [5].
- **Complex Networks**: Sharma–Mittal entropy quantifies node diversity and structural heterogeneity [8].
- **Ecology**: Rényi and Tsallis entropies measure species diversity [9].

6. Discussion

While Shannon entropy is fundamental for independent events, generalized entropies provide flexibility for correlated or heavy-tailed systems. Rényi emphasizes dominant probabilities, Tsallis captures non-extensivity, and Sharma–Mittal unifies these approaches. Selecting an appropriate entropy measure depends on system properties and analytical goals.

7. Conclusion

This study compared Shannon, Rényi, Tsallis, and Sharma–Mittal entropies in terms of definition, properties, and application. Generalized entropies are crucial for modeling complex systems, offering tunable sensitivity and non-additive behavior, which classical Shannon entropy cannot capture. Future work may explore further applications in machine learning, complex networks, and ecological modeling.

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