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Advances in Copula Modeling: Review

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Abstract: Here, we reviewed numerous works of different authors on Copula modeling. Theoretical developments and applications of Copula modeling in various research papers including past as well as ongoing research work were reviewed and discussed in this paper. In this paper, we mainly focused on the distributional and different properties of Copula and also gave much importance to study the practical applications of Copula modeling in various fields. Also here, we illustrate a real data interpretation on the data representing consumer price index from the year 2013 to 2024 of Rural and Urban areas India.

Keywords: Copula, Copula estimation, Copula modeling

1. Introduction

Copula Modelling is a method that is often applied to study linkages between variables.
By applying copula modeling, which is especially useful in risk and survival analysis, the study of tail dependencies is possible. Copula modeling is especially applicable to economic and financial modeling because it can be applied to forecast financial contagion and "boom" or "bust" periods. The dependencies within the modeled dataset and the extreme events that could be present in the dataset tails can be modeled by applying different copulas that are available in bivariate copula modeling. Because the two different modeling methods, one of which is non time series and the other is time series, may not be able to completely capture the complexity of the types of copulas that are found in the literature, financial copula modeling tends to diverge.

In recent years, the use of copulas in data modeling has become popular because it enables one to choose suitable marginal distributions for the different components of a multivariate system. Any multivariate distribution function can be used as a copula. Copulas play a significant role in constructing statistical models, and this is why new copulas have been developed. The objective of the above studies is to develop more flexible families of bivariate (multivariate) distribution functions with interesting properties such as symmetries, tail dependencies, and wide ranges of association.

Archimedean copulas are widely employed in empirical modeling and have drawn a lot of interest because of their ability to model interdependence between exchangeable random variables and their computational tractability. A generator determines an Archimedean copula in a unique way (up to a scalar multiple). By providing all the information regarding the multi-dimensional dependence structure the univariate generator simplifies the analysis of a multivariate copula into a single univariate function. There are several ways to quantify the dependency or association between random variables and the copula is one that can capture the scale invariant or distribution free character of the association between random variables.

An observational sequence on one or more variables across time is called a Time series. Because previous events have the power to shape future happenings time is a crucial dimension. Time series are characterized by two primary features: autocorrelation or the association between measurements of the same variable across successive time intervals and data frequency. In order to employ properly described residuals in the copula model these traits need to be carefully modeled.

2. Review and its Methodology

Olivier Scaillet and Jean-David Fermanian (2004) discussed some possible statistical hazards when modeling cross-dependencies in financial applications using copulas. They examined particular issues in estimate, empirical copula selection and the design of time-dependent copulas. The issues that come up in estimate empirical copula selection and the design of time-dependent copulas are given particular consideration.

Pranesh Kumar and Mohamed M Shoukri (2007) examined that the standard correlation based prediction models are shown to be inappropriate for modeling dependence in populations with asymmetrical tails and copula based prediction modeling is shown to be a suitable substitute. To validate the proposed copula based prediction model independent bootstrap samples were used.

David Gunawan *et al.* (2016) defined the likelihood as the density of the observations with respect to a mixed measure extending the research on copulas with discrete or continuous marginals to the situation when some of the marginals are a combination of discrete and continuous components. Specifically, they concentrate on Gaussian and Archimedean copula mixes. Using Markov chain Monte Carlo for estimate the inference is Bayesian. By using the algorithms and techniques to estimate a multivariate income dynamics model they demonstrate them.

Paula V. Tófoli *et al.* (2017) presented a novel method that combines the Markov switching model with time-varying copulas to explain the dependence between global financial returns over time. To the return data of the FTSE-100, CAC-40, and DAX indices, they apply these copula models in addition to those released by Patton (2006), Jondeau and Rockinger (2006), and Silva Filho, Ziegelmann, and Dueker (2012). They are also curious about how these approaches compare in terms of the dynamics of reliance that occur and how well the models predict potential capital losses. The models are compared and chosen based on VaR estimates because risks associated with extreme occurrences are significant for risk management.

N. Unnikrishnan Nair *et al.* (2018) explained the qualities of dependability function analogues that are represented in terms of copulas. The copulas of a bivariate exponential family of distributions are used as an example to demonstrate the findings of the study. They release some basic definitions and findings that help in modeling lifetime data using bivariate copulas instead of the usual approach of using bivariate distributions. It was found that the mean residual quantile function and the copula-based hazard rate have some advantages over their distribution-based counterparts.

M. Ishaq Bhatti and Hung Quang Do (2019) analyzed in depth the development of copula models and their applications in the areas of environmental sciences, forestry, fuel cells, and energy.

Copulas have been applied in many energy, environmental, and forestry studies, particularly when using hydrology to estimate the correlations between tree height, diameter, volume, water flow, and energy generation. Copula models have also been widely applied in hydrometeorology, such as earthquake prediction, wind speed, irrigation, energy prediction, and rainfall prediction.

Nur Amirah Buliah and Wendy Ling Shin Yie (2020) analyzed the rainfall volume and duration using a copula for various stations.This study have three specific goals, the first one is to estimate the correlation between rainfall volume and duration and the second one is to fit rainfall volume and duration into a copula; and finally the third one is to determine the best-fitting copula.The suggested approach is used to analyze hourly rainfall data from a few chosen stations in Terengganu, Selangor, Kuala Lumpur and others. The suggested method has the advantage of offering a more adaptable way to model multivariate distributions for which the volume and duration marginal distributions do not always belong to the same distribution family. Generally speaking, the copula can accurately model the amount and duration of rainfall.

Gery Geenens (2020) investigated the viability of copula modeling for discrete data again. It offers a simpler framework that makes copula concepts transfer naturally to the discrete situation. In actuality it is an effort to revive some of Udny Yule's century-old concepts as he had previously described a construction akin to copulas.Lastly they point out that applying the bivariate ideas discussed in this study to higher dimensions is simple. The useful characteristics of higher-order odds ratios particularly margin-freeness are retained.

Paul R. Dewick and Shuangzhe Liu (2022) examined normal copula modeling and financial copula modeling and explained why time series and non time series copula modeling employ different methodologies. Here, normal copula modeling and financial copula modeling both follow the same process but financial copula modeling requires that the time series be properly defined. Once the time series has been identified the copula model may be generated and the uniform marginal distribution can be simulated using a pseudo-CDF. The time series components need to be appropriately described in order to produce a correctly stated copula model

Andreas Tsanakas and Rui Zhu (2022) examined copula model selection as a challenge related to picture recognition. They contributed model selection methods that integrate image features with statistical data and they use the pre trained AlexNet to extract image features from heatmaps. A Support Vector Machine classifier is used along with dimension reduction by Principal Component and Linear Discriminant Analyses. Simulation experiments demonstrate that the copula model selection job is more accurate when image data is used especially when sample sizes and correlations are small. This result suggests that transfer learning can help with statistical model selection processes. They show how to apply the suggested method to the joint modeling of the RISX and MSCI indices weekly returns.

Shixiang Gu et al. (2023) developed a vine copula model that included rolling judgments and real-time prediction result correction in order to simulate and forecast lake water level. The model was applied to predict the water levels of Erhai Lake in the short and long term on the Yun-gui Plateau in southwest China. With the increase in the number of factors, the accuracy of the predictions increased, and the results showed that the daily water level prediction accuracy was higher than the monthly prediction accuracy. In addition, among the three models, the vine copula model yielded the highest accuracy in the nonlinear relationship between the predicted water level and climatic factors, especially during the rainy season, compared with the support vector regression and back-propagation neural network models. The accuracy of the vine copula model in prediction decreased with smaller sample numbers and without runoff data. The relative errors of the prediction accuracy of less than 5%, 10%, 15%, and 20% increased to 70%, 83%, 95%, and 98%, respectively, due to improved analysis of the model errors.

Zhenlong Chena *et al.* (2023) investigated the relationships between actual Chinese industries and the effect of dynamic connections on portfolio risk optimization they build a dynamic factor copula-mean-ES model. They reached a conclusion that the risk in portfolio optimization can be properly measured by the three dynamic factor copula models constructed in this study. In particular, through the proper description of the dependent relationship between China's actual industries, the dynamic heterogeneous factor copula model has the highest accuracy in calculating the minimal ES. This indicates the good knowledge acquired about the role of dynamic dependency relations in portfolio risk optimization. Then they concluded that the public utilities industry is given the highest weight in the analysis of portfolio risk optimization. This is because it contains industries that have a significant influence on the development of the national economy, such as electricity, heat power, and water.

On the other hand, the mining industry gives the lowest weight because it is very dependent on steel and non-ferrous metals, and it is also exposed to higher risks due to the rapid development of new energy sources. Since financial information is shared, the model has a promising relevance for future economic studies despite the fact that this study only focuses on China and does not include information from other countries.

Dimuthu Fernando *et al.* (2024) recommended a bivariate count time series model that was constructed with the use of copula theory. The candidate copula family to represent both serial dependence and cross correlation between the two time series is the Gaussian copula. The model is equally effective at simulating negative and positive cross correlations. The likelihood-based estimate approach with importance sampling is evaluated using simulated cases to assess the bivariate normal integral. Temperature and humidity bivariate counts are examined to demonstrate the efficacy of the suggested model.

Jonas Asplund and Arkady Shemyakin (2025) suggested distinct Autoregressive Integrated Moving Average (ARIMA) models for the analysis of excess mortality for multiple nations. They proposed the use of Bayesian pair copula selection for vine copulas in the joint excess mortality model. Weekly death rates from 2019 to 2022 in the US, Canada, France, Germany, Norway, and Sweden are examined in this study. The proposed ARIMA models have negligible residual autocorrelation and low lags. Only Norway's residuals displayed normalcy. As a fair fit the others proposed skewed Student-t distributions. A vine copula model was then used to represent the relationship between the ARIMA residuals for different countries with the geographically most distant countries exhibiting little to no correlation. The validity of the fitted distributions and the resulting vine copula was investigated using data from 2023. According to goodness of fit testing the vine copula employed was legitimate and the fitted distributions were appropriate with the exception of the USA. They came to the conclusion that the COVID-19 excess mortality time series models are feasible. All things considered, the proposed methodology is appropriate for developing collaborative pandemic mortality projections for many nations or geographic areas.

3. Real data Application

We illustrate a real data interpretation on the data representing consumer price index from the year 2013 to 2024 of Rural and Urban areas India. The plot of the two generated time series shows a general upward trend for both variables from 2013 to 2024, reflects a gradual increase over time. Although both series exhibit short-term fluctuations due to random variations, the overall movement remains positive. Series B consistently lies above Series A, indicating relatively higher values throughout the period. The similarity in their movements suggests that the two series may share a common long term growth.

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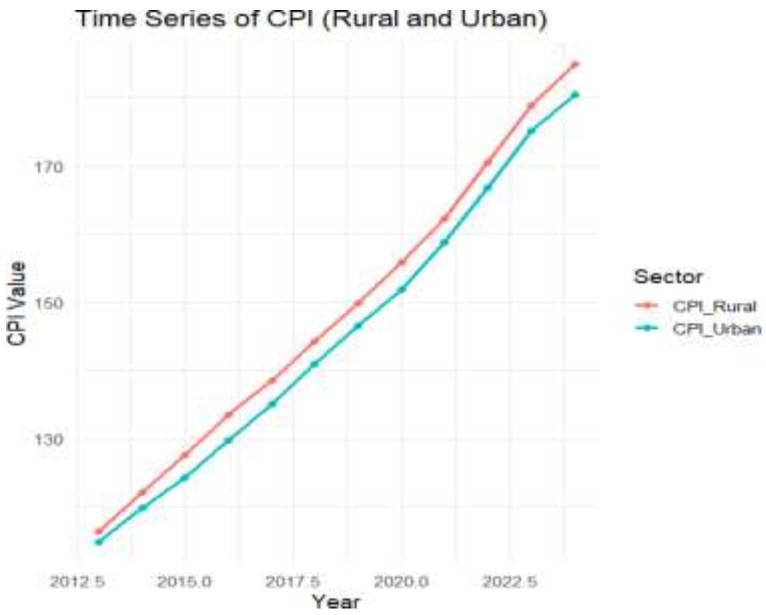

Figure 1: Time Series plot of CPI

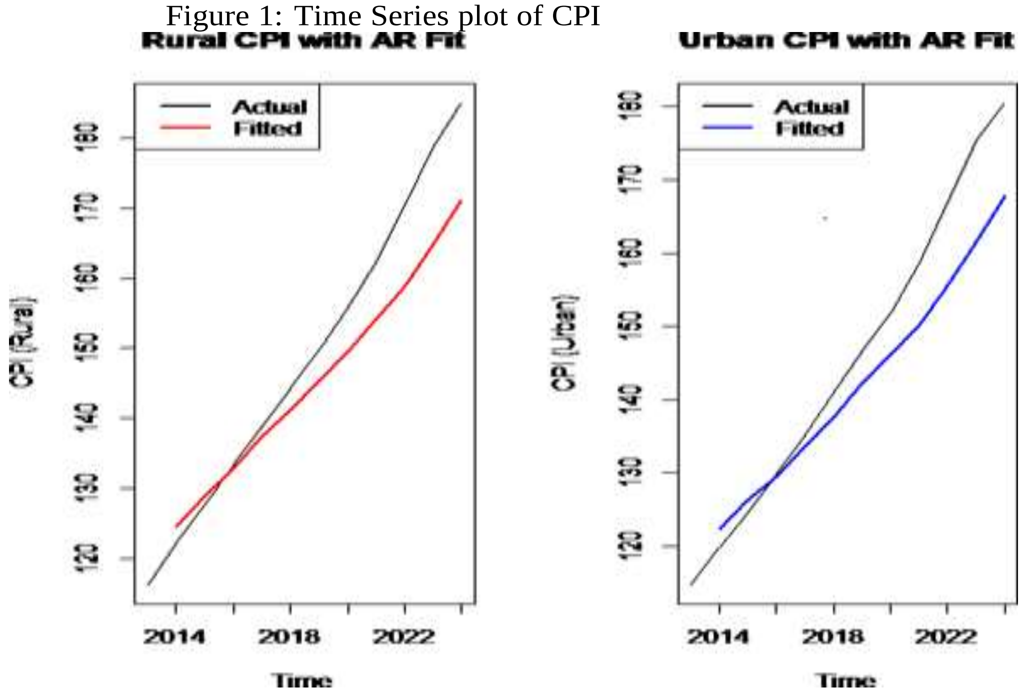

Figure 2: AR Fit plot

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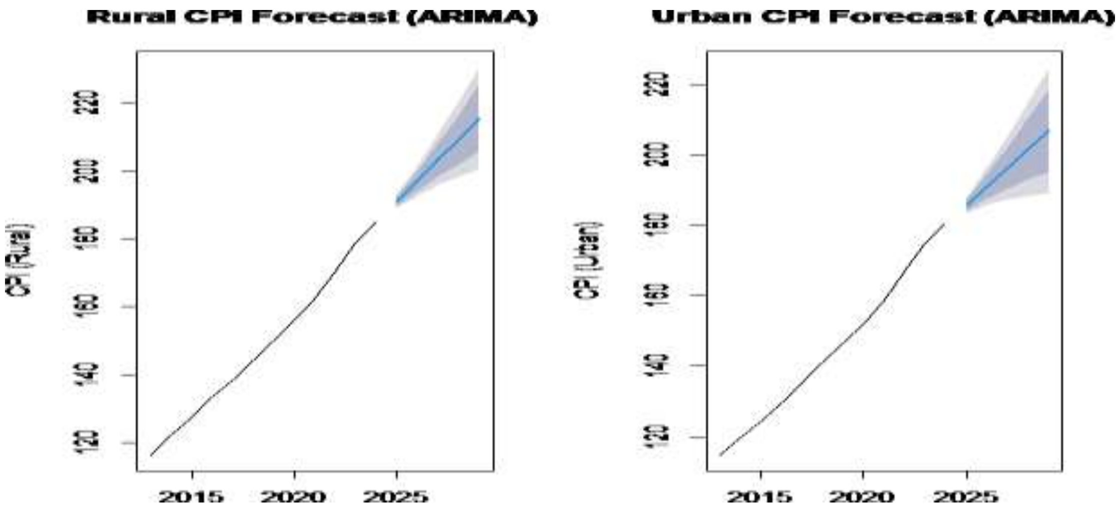

Figure 3: Forecast values

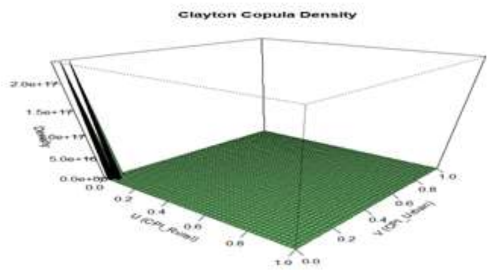

Figure 4: Clayton Copula density

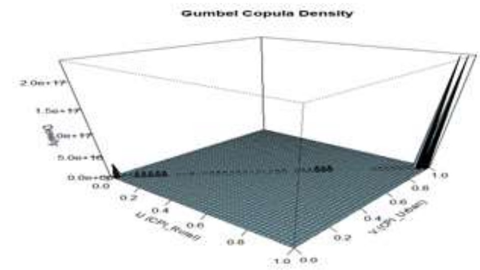

Figure 5: Gumbel Copula density

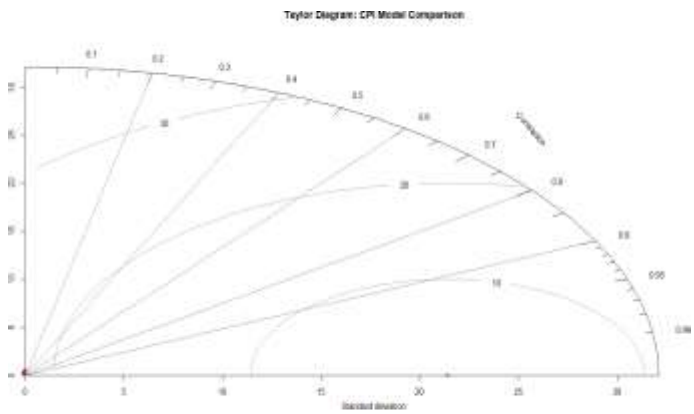

Figure 6: Taylor diagram of Clayton Copula

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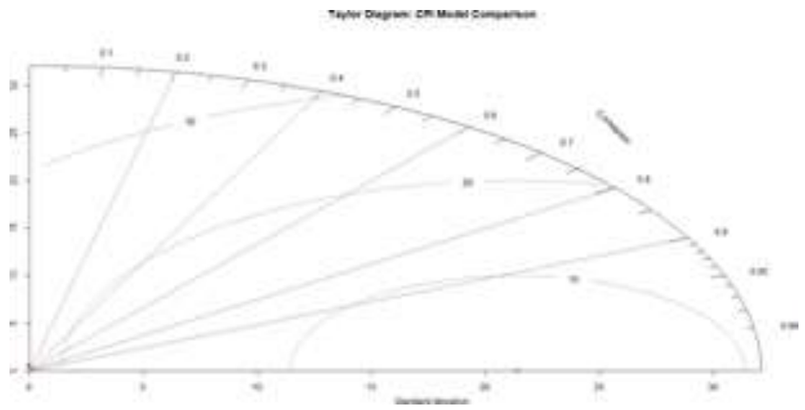

Figure 7: Taylor diagram of Gumbel copula

3.1. Interpretation

- The Rural CPI's AR(1) coefficient is 0.7447, which indicates strong persistence in Figure 1,. This means that rural inflation fluctuates gradually and is heavily impacted by the CPI from the prior year. The urban CPI, on the other hand, has an AR(1) coefficient of 0.6923, indicating that although it is still persistent, it responds to new price adjustments faster than the rural sector.
- In Figure 2, the plots illustrate the AR(1) model fit for both Rural and Urban Consumer Price Indices (CPI) over time (from 2014 to 2023) Here both rural and urban CPI statistics fit well with the AR(1) model, which captures the inflation dynamics of both over time. Both sectors show a high degree of persistence, indicating that present inflation trends are strongly influenced by previous price levels. The Rural CPI, on the other hand, exhibits somewhat greater persistence (AR(1) = 0.7447), suggesting slower and more gradual changes in rural inflation. On the other hand, the Urban CPI (AR(1) = 0.6923) reacts to new price adjustments faster even if it is still persistent. Overall, both sectors' inflation patterns are steady and predictable, with rural inflation showing more traction than previous trends.
- Based on the rising trajectories in the ARIMA estimates inflationary pressure is expected to persist in both rural and urban sectors in the upcoming years. The rural CPI projection indicates a little stronger inflation momentum which is consistent with its higher AR(1) coefficient that was previously found even if both trends exhibit persistence which is explained in in Figure 3,. The expanding confidence bands show that although inflation is predicted to

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continue rising the precise rate of price changes may be influenced by future volatility and the state of the economy.

- In Figure 4, The Clayton copula is known for strong lower tail dependence — it models the association between joint low values of two variables. Here the plot confirms that the density "spike" in the lower-left corner shows strong co-movement when both indices fall. The upper-right (high CPI values) region is almost flat which means weak upper tail dependence. Inflation (CPI) tends to be strongly coordinated downward in urban regions when it is low in rural ones. However their link somewhat deteriorates amid high inflation as one may increase more quickly than the other. As a result there is a stronger correlation between the rural and urban CPIs during times of low inflation than during times of high inflation. This pattern indicates that asymmetry particularly dependency during times of low CPI is captured by the Clayton copula. Clayton does well if we want to model joint low infla- tion or deflation risk. A Gumbel or Frank copula may be more appropriate for co-movements with strong inflation.
- The Archimedean copula family includes the Gumbel copula. So in Figure 5, It is intended to mimic the tendency for large (high) values in one variable to be connected with large values in another, a phenomenon known as upper-tail dependence. In our instance, it illustrates whether high rural and urban CPIs frequently coexist. In upper tails or during times of high inflation the Gumbel copula density demonstrates a considerable co-movement between the rural and urban CPIs. This suggests that price increases in rural areas frequently accompany inflation shocks in urban areas maybe as a result of pressures on the shared supply chain rising gasoline prices or rising food prices. Their link is looser when inflation is low because the lower tail reliance (when both indices are low) is less. AIC of clayton copula is -770 and the AIC of Gumbel copula is -853.375 Here the AIC of Gumbel copula is low, so Gumbel copula is better than Clayton copula.
- In figure 6, the Clayton Copula for our Rural–Urban CPI data is displayed in the Taylor diagram captures co-movement when the CPI is low or moderate (low inflation, price stability) . It performs worse at times of high CPI (inflation increases). Overall, it might be marginally off in the higher tail but closer to the reference point along low CPI ranges. Clayton prioritizes stable inflation co

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movement over extremely high inflation, in contrast to Gumbel (upper-tail reliance).

- The Taylor diagram which is shown in Figure 7 is that when inflation is high, the Gumbel copula fits the Rural–Urban CPI data better than Clayton copula. It captures co-movement during inflation peaks and has a stronger connection with observed data.The film accurately simulates volatility because its standard deviation is around the reported CPI. A comparatively lower RMSE suggests generally strong performance. In contrast to Clayton its fit could marginally deteriorate during times of low CPI.

Finally, Rural and urban CPI move very closely together during inflationary times according to the Gumbel copula Taylor diagram which gives a more realistic upper-tail (high CPI) dependence structure than Clayton.

4. Future Scopes

We can create copulas with conditional independence as well as we can use deep neural networks to learn copula functions. Copulas can be used to automatically create features based on dependency hierarchies. We can create algorithms for updating copula parameters in real time. We can employ copulas at various spatial resolutions. We can use it for robust high-dimensional estimation. Copulas can be used to create artificial datasets that strike a balance between realism and justice.

5. Conclusion

From a specialized statistical idea, copula models have developed into a potent instrument that is frequently used in disciplines including environmental research risk forecasting, finance, and epidemiology. Copulas provide a more dependable framework for modeling complicated relationships by overcoming the shortcomings of conventional correlation-based models in capturing asymmetric and nonlinear dependencies. Value at Risk (VaR) and other financial risk management applications have been enhanced by developments like time-varying and regime-switching copulas, and research on the environment have demonstrated how well they capture interactions between eco- logical, hydrological, and meteorological systems. The interpretability and predictive accuracy of multivariate models have been improved by developments such as dynamic factor copulas, vine copulas, and real-time updating techniques.Additionally, copulas exhibit promise in study on global health, including the ability to forecast excess mortality during pandemics. Copula models are expected to become more and more important in high-dimensional predictive modeling and decision-making as approaches continue to incorporate Bayesian and machine learning techniques.

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